

$$\text{Estimate } I = \int_{-\pi/2}^{\pi/2} [\text{sinc}(x)]^{300} dx$$

This could be a difficult integral to do exactly, but it is possible to approximate. $\text{sinc}(x) = \sin x / x$, and so the principal contribution to the integral will come from small x : $[\text{sinc}(x)]^{300}$ will decay like x^{-300} . Therefore, we may use a small angle approximation

$$\begin{aligned} [\text{sinc}(x)]^{300} &= \left[\frac{\sin(x)}{x} \right]^{300} \\ &\approx \left[1 - \frac{x^2}{6} + \dots \right]^{300} \quad \{|x| \ll 1\} \end{aligned}$$

Since we are interested in small x^2 , and because 300 is a large exponent, we can further approximate the integrand as

$$[\text{sinc}(x)]^{300} \approx \exp(-50x^2) \quad \{|x| \ll 1\}$$

Our integral is then

$$I \approx \int_{-\pi/2}^{\pi/2} \exp(-50x^2) dx$$

Since we only care about small x (large x contributes negligibly), we can extend the limits

$$I \approx \int_{-\infty}^{\infty} \exp(-50x^2) dx$$

This is a standard integral with solution

$$\begin{aligned} I &= \sqrt{\frac{2\pi}{2 \times 50}} \\ &= \frac{\sqrt{2\pi}}{10} \end{aligned}$$

We just need to estimate this

$$\begin{aligned} I &\sim \frac{1.4 \times 1.8}{10} \\ &\sim \frac{2.5}{10} \\ &\sim \underline{0.25} \end{aligned}$$

[Exact: $I = 0.2505374638056364\dots$; $\sqrt{2\pi}/10 = 0.25066282746310004\dots$]