Estimate the density of the Sun.

Being an astronomer, I know the mass of the Sun is $M_\odot \simeq 2 \times 10^{30}$ kg and the solar radius is $R_\odot \simeq 7 \times 10^8$ m. Hence the density is

$$\rho_\odot \simeq \frac{3}{4\pi}\frac{M_\odot}{R_\odot^3}$$
$$\simeq \frac{3}{4\pi}\frac{2\times10^{30}}{(7\times10^8)^3}$$
$$\simeq \frac{1}{2}\frac{10^6}{350}$$
$$\simeq 1/7 \times 10^4$$
$$\underline{\simeq 1.4 \times 10^3 \text{ kg m}^{-3}}$$

The Sun is 1.4 times the density of water.

It's not too fun knowing the answer: can I estimate $M_\odot$ and $R_\odot$? I know the distance to the Sun is

$$r \simeq 8\tfrac{1}{3} \text{ light-minutes}$$
$$\simeq 500 \text{ light-seconds}$$

and the angular size of the Sun is about the same as the Moon, with diameter $\Theta \simeq 0.5°$. Hence

$$R_\odot \simeq \tfrac{1}{2}\Theta r$$

The mass can be worked out from the Earth's orbital period: one year is

$$T \simeq 2\pi\sqrt{\frac{r^3}{GM_\odot}}$$
$$\Rightarrow M_\odot \simeq 4\pi^2 r^3/GT^2$$

Since I know the length of a year (about $\pi \times 10^7$ s) and the various constants I can work everything out

$$\rho_\odot \simeq \frac{3}{4\pi} \cdot \frac{4\pi^2 r^3}{GT^2} \left(\frac{2}{\Theta r}\right)^3$$

$$\simeq \frac{24\pi}{G\Theta^3 T^2}$$

We see that r cancels, so we didn't need that!

$$\rho_\odot \simeq \frac{24\pi}{(6.67\times10^{-11})(0.5\times \pi/180)^3(\pi\times10^7)^2}$$

$$\simeq \frac{24\times360^3}{(20/3)\pi^4\times10^3}$$

$$\simeq \frac{3.6^4\times10^7}{10^6}$$

$$\simeq 4^4(1-1/10)^4\times10$$

$$\simeq 256(1-4/10)\times10$$

$$\simeq \underline{1500 \text{ kg m}^{-3}}$$

This is a little high, but not too bad at all.

[Exact: 1410.9 kg m^{-3}]