Estimate the number of musical instrument repairers and tuners in the Greater New York metropolitan area.

This is a variant of the classic Fermi problem. It requires estimating several different quantities and combining them to produce an answer. I could find it difficult to just guess the number, so I will piece it together, but I will first try to guess a plausible range. What could be a reasonable lower bound? Let's imagine there was just one highly efficient company. I'd imagine that a small company might employ 20 people. So this seems like a sensible lower bound.

Now, how about an upper bound? I think that a small town could just about support one repairer. A small town might have a population of 10,000. That would mean 1 in 10,000 people is a repairer. I could guess it can't be too much higher than that as I don't know anyone who's employed in the sector – that would be odd if it were, say, 1 in 1,000.

To complete our upper bound we could need to know the population P of New York. I know it is a massive city, one of the biggest in the world. Greater London has a population of about 10 million I think. New York could be bigger, say 15–20 million. I think that the LA conurbation is the largest in the US, and I don't think this is much over 20 million, if at all, so perhaps

$$P \simeq 18 \times 10^6 \text{ people.}$$

The total population of the US is about 300 million, so my estimate gives ~13% of the population living in the two largest urban areas. That doesn't seem bad.

With $P = 1.8 \times 10^7$ people we could guess 1800 repairers using our 1 in 10,000 figure. This must be an upper bound as it was based of a small town (I wouldn't be surprised if it was more like 1 in 20,000 as an upper bound). We could also expect a higher efficiency in a big city (less travel distance) and more competition

so we could expect the actual number to be lower.

Having set our plausible range as ~20 - 1,800 we can work on a more accurate value. Let us consider the demand for tuning or repairing and try to match that to the number of people that could support. If there are N jobs per year, a repairer can do n jobs a day and works d days a year, then the number of repairers would be

$$x = \frac{N}{nd}$$

I think that tuning a piano may take an hour and it might take an hour getting there and back. Repairing another instrument seems like it could take longer, but people would probably take them to a shop. We'll ignore small jobs as they wouldn't make much money or perhaps could be done by a music shop by someone who isn't employed as a repairer full time. Let us say that $n = 4\ \text{day}^{-1}$, which is reasonable for an eight-hour day.

Assuming a repairer works five days a week and gets two weeks holiday a year (Americans don't get much holiday),

$$d = 5 \times 50 = 250\ \text{days yr}^{-1}$$

To estimate N, we'll start again from the population P, if there are three people on average per household (families might be four people, but not all households are families) then there are

$$H = \frac{1.8 \times 10^7}{3} \simeq 6 \times 10^6\ \text{households.}$$

I think on average there may be one instrument for every two households — from my experience that seems about right, keeping in mind musical families tend to have more instruments, so we need to distribute evenly. The number of instruments is

$$I \simeq \frac{6\times10^6}{2} \simeq 3\times10^6 \text{ instruments}$$

$I/p = 1/6$ instruments per people, which doesn't seem too far from my experience. Next, I think 10% of instruments may be played regularly. If they are played regularly they are probably well looked after, the owner will probably do minor things themselves, so they might need tuning every other year. Those which are not played regularly may get tuned every ten years. These seem to imply that there is one repair per ten instruments a year

$$N \simeq \frac{3\times10^6}{10} \simeq 3\times10^5 \text{ yr}^{-1}$$

That gives $N/p \simeq 1/60$ repairs per year per person. This means we can safely ignore repairs and tunings of instruments in places like churches, schools or concert halls, which would be more frequent (as the instruments are heavily used), but will be much more dilute per capita, maybe accounting for one for a few hundred people.

Putting everything together

$$x \simeq \frac{3\times10^5}{4\times250}$$
$$\simeq \frac{3\times10^5}{10^3}$$
$$\simeq 300$$

That is in the middle of our sensible range, so it seems safe.
$x/p \simeq 1/60{,}000$, which is a factor of six higher than our estimate based off a small town. That seems reasonable. Cambridge has a population of about 100,000, so if the scaling were about the same, there may be two repairers. Again that is believable. It seems like we have arrived at a good estimate.

[Exact: $x = 380$; $P = 18,897,109$ (2010),]