

Estimate the mass that could be lifted by the Hindenburg: the balloon had a volume of  $2 \times 10^5 \text{ m}^3$ .

Archimedes' principle tells us that the lifting force of a balloon is equal to the difference in weight between the balloon and the displaced fluid

$$L = W_{\text{air}} - W_{\text{balloon}}$$

To convert to masses we just need to divide by the gravitational acceleration  $g$ :

$$\begin{aligned} m_{\text{lift}} &= m_{\text{air}} - m_{\text{balloon}} \\ &= (\rho_{\text{air}} - \rho_{\text{balloon}})V \end{aligned}$$

where we have changed to densities and volumes. We know that the balloon was filled primarily with hydrogen gas. Let us only consider this. We can calculate the density from the ideal gas law

$$\rho = \frac{\mu P}{kT}$$

where  $\mu$  is the molecular mass,  $P$  is the pressure and  $T$  is temperature. I will assume atmospheric pressure and temperature of about  $0^\circ\text{C}$ , remembering the zeppelin will travel at some altitude and not in a lab: that is  $P \approx 10^5 \text{ Pa}$ ,  $T = 273 \text{ K}$ . I know the mass of  $\text{H}_2$  is  $\mu_{\text{H}_2} = 2u$ , where  $u = 1.66 \times 10^{-27} \text{ kg}$  is the atomic mass unit, and Boltzmann's constant is  $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$ . I also know the density of air is about  $\rho_{\text{air}} = 1.3 \text{ kg m}^{-3}$ , so I don't need to work that out. Using these

$$\begin{aligned} m_{\text{lift}} &\approx \left( \rho_{\text{air}} - \frac{\mu_{\text{H}_2} P}{kT} \right) V \\ &\approx [1.3 - (2 \times 1.66 \times 10^{-27}) (10^5) / (1.38 \times 10^{-23}) (273)] (2 \times 10^5) \end{aligned}$$

$$\approx \left( 1.3 - \frac{2 \times 10^5 / 3 \times 10^{-22}}{4/3 \times 10^{-23} \times 273} \right) (2 \times 10^5)$$

$$\approx \left( 1.3 - \frac{10^2}{10^3} \right) (2 \times 10^5)$$

$$\approx \underline{2.4 \times 10^5 \text{ kg}}$$

[Exact:  $m_{\text{diff}} = 232,000 \text{ kg}$ ]