Estimate the time difference between actual time and that of a pendulum clock which has been kept at a temperature 6K higher than its calibration temperature, where its period was $P = 1s$, after 1 year:

The period of a pendulum is

$$P = 2\pi\sqrt{\frac{L}{g}}$$

where L is length and g is gravitational acceleration. This can be got on dimensional grounds up to a factor of 2π. Length varies with temperature

$$L = L_0(1 + \alpha\Delta T)$$

where L_0 is the length at $\Delta T = 0$ and α is the thermal expansivity. I know $\alpha = 16.5 \times 10^{-6} K$ for copper, so I will use this as an estimate. We have

$$P(T) = 2\pi\sqrt{\frac{L_0(1+\alpha\Delta T)}{g}}$$

with $P_0 = 1s$.

Let n be the number of oscillations of the pendulum, and Y be the number of seconds in a year. The time difference is just the difference in the number of oscillations (since the period should be 1s).

$$\begin{aligned}
\Delta\tau &= \Delta n \\
&= \frac{Y}{P_0} - \frac{Y}{P(T)} \\
&= \frac{Y}{P_0}\left(\frac{P - P_0}{P}\right) \\
&= Y\left(1 - \frac{1}{\sqrt{1+\alpha T}}\right)
\end{aligned}$$

Since $\alpha\Delta T$ is small we can binomially expand

$$\Delta\tau \approx Y\left[1 - \left(1 + \frac{\alpha\Delta T}{2}\right)\right]$$
$$\approx -\frac{\alpha\Delta T Y}{2}$$

The minus sign indicates the pendulum clock is slow, but we only really care about the magnitude here. Inserting values using $Y \approx \pi \times 10^7 s$

$$|\Delta\tau| \approx \frac{16.5\times10^{-6}\times 6\times\pi\times10^7}{2}$$
$$\approx \frac{100\times\pi\times10}{2}$$
$$\approx 1.57\times10^3 s$$

Note that we never needed the 2π we couldn't get from dimensional analysis.

[Exact: $|\Delta\tau| = 1562 s$ assuming copper]