

Estimate the ratio of frequencies in helium and air of sound from an instrument of fixed size:

Since wavelength is fixed ratio of frequencies is ratio of sound speeds

$$\frac{v_{He}}{v_{Air}} = \frac{c_{He}}{c_{Air}}$$

I know c_{Air} ; to work out c_{He} I use

$$c = \sqrt{\frac{\gamma P}{\rho}}$$

assuming atmospheric pressure could seem sensible, since helium is monatomic $\gamma_{He} = 5/3$, but still need density. Assuming ideal gas laws holds

$$\begin{aligned} \rho &= mn \\ &= \frac{mP}{kT} \end{aligned}$$

where n is number density and T is temperature. The mass of helium is $m_{He} = 4m_0$.

$$c_{He} = \sqrt{\frac{\gamma kT}{m_{He}}}$$

Thus, inserting values

$$\begin{aligned} \frac{v_{He}}{v_{Air}} &= \frac{1}{c_{Air}} \sqrt{\frac{\gamma_{He} kT}{m_{He}}} \\ &= \frac{1}{330} \sqrt{\frac{5/3 (1.38 \times 10^{-23}) (300)}{4 (1.67 \times 10^{-27})}} \\ &= \frac{1}{330} \sqrt{\frac{5/3 \times 4/3 \times 3 \times 10^{-21}}{4 \times 5/3 \times 10^{-27}}} \end{aligned}$$

$$\approx \frac{1 \sqrt{10^6}}{330}$$

$$\approx \frac{1000}{330}$$

$$\approx \underline{3}$$

[Exact: $v_{\text{Hg}}/v_{\text{air}} = 2.89$ assuming $T = 20^\circ\text{C}$ and relative humidity of air 82.3%]