

Estimate the mass of the Moon:

To find the Moon's mass we need to know its size. I know that during an eclipse the Moon covers the Sun, so they have the same angular diameter, and I know the Sun's diameter is  $\Theta \approx 0.5^\circ$ . The Moon's radius is then

$$r = (\Theta/2)R$$

where  $R$  is the Moon's distance from the Earth. Since I know the Moon's period is  $T = 28 \text{ days} \approx \pi \times 10^7 / 13 \text{ s}$  and the mass of the Earth is  $M_\oplus = 6 \times 10^{24} \text{ kg}$ , I can work out  $R$  from Kepler's third law

$$\begin{aligned} \frac{GM_\oplus}{R^3} &= \left(\frac{2\pi}{T}\right)^2 \\ R^3 &= \left(\frac{T}{2\pi}\right)^2 GM_\oplus \end{aligned}$$

The mass of the Moon is

$$\begin{aligned} M &= \frac{4\pi r^3}{3} \rho \\ &= \frac{4\pi \Theta^3 R^3}{3 \cdot 8} \rho \\ &= \frac{\pi \Theta^3}{6} \left(\frac{T}{2\pi}\right)^2 GM_\oplus \rho \end{aligned}$$

We need an estimate for the density  $\rho$ : I know quartz has density  $\rho_{\text{quartz}} = 2650 \text{ kg m}^{-3}$ , the Earth has density

$$\begin{aligned} \rho_\oplus &= \frac{3M_\oplus}{4\pi R_\oplus^3} \\ &= \frac{3 \times 6 \times 10^{24}}{4\pi (6.4 \times 10^6)^3} \end{aligned}$$

$$\begin{aligned}
&= \frac{18 \times 10^{24}}{4\pi (2^6 \times 10^5)^3} \\
&= \frac{18 \times 10^{24}}{\pi (2^{18})^2 \times 10^{15}} \\
&= \frac{18 \times 10^9}{\pi \times 10^6} \\
&= 6 \times 10^3 \text{ kg m}^{-3}
\end{aligned}$$

The Moon will have a density somewhere inbetween: quartz is a light mineral, but the Moon doesn't have the Earth's iron core. I think the density would be closer to quartz, so let's guess  $\rho \sim 3.5 \times 10^3 \text{ kg m}^{-3}$ .

$$\begin{aligned}
M &= \frac{\pi \left(\frac{\pi}{6}\right)^3 \left(\frac{10^7}{26}\right)^3 (6.67 \times 10^{-11}) (6 \times 10^{24}) (3.5 \times 10^3)}{6 \left(\frac{\pi}{360}\right)^3 \left(\frac{10^7}{26}\right)^3} \\
&= \frac{\pi^4 \times 40 \times 3.5 \times 10^{30}}{6 \times 3.6^3 \times 2.6^2 \times 10^6} \\
&= \frac{10^2 \times 4 \times 10^{23}}{6 \times 3.6^2 \times 2.6^2} \\
&= \frac{4 \times 10^{25}}{6 \times 13 \times \frac{25}{4}} \\
&= \frac{16 \times 4 \times 10^{23}}{80} \\
&= 8 \times 10^{22} \text{ kg}
\end{aligned}$$

[Exact:  $M = 7.3483 \times 10^{22} \text{ kg}$ ;  $\rho = 3.345 \times 10^3 \text{ kg m}^{-3}$ ]